

Stiffness Requirements for Longitudinal Stiffeners of Trapezoidal Box Girder Bottom Flanges

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Abstract

The reason for the widespread use of steel box girders is that they have high structural efficiency due to the high bending, high torsional stiffness and rapid erection. For bottom flange of the girders, the buckling behavior during production and erection due to compression strength can be a problem. The compression plate with longitudinal stiffeners typically renders an economic. The optimal design of longitudinal stiffeners is discussed. The results are based on 3-D FEA (ANSYS19.2) of many stiffened compression bottom flange models, the moment of inertia requirement of bottom flange longitudinal stiffener is investigated by studying the effect of many parameters as longitudinal stiffeners numbers, stiffener sections, plate aspect ratio and compression flange thickness. Also, the stiffeners effect on the compression panel plate stresses were studied by modeling girder with and without longitudinal stiffeners. The finite element method is useful as they can be used to study the plate with stiffeners in an economical way, and we don't need experimental and laboratory tests.

Keywords: Box girder bridge, Longitudinal stiffeners, Finite element modeling, ANSYS program.

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1. Introduction

Due to the large bending and torsional stiffness of steel box girder as well as rapid erection they are used in many structural applications. The local buckling strength of the flanges and webs could have a negative effect on the load carrying capacity of steel box girder. More importantly, the designated longitudinal stiffener size depends on the buckling action which caused by the compression on the bottom flange of the girder. Several researches have been done in Europe, Japan, etc. After a number of disastrous collapses of many box girders bridges that happened during the erection of bridges located in Austria (Fourth Danube Bridge in 1969), Australia (West Gate Bridge in 1970), Wales (Milford Haven Bridge in 1970) and Germany (Koblenz Bridge in 1971).

Highly complex analytical and theoretical research results on the stiffened bottom flanges were presented in 1952 by Bleich [1] and Timoshenko and Gere in 1961 [2].

Chatterjee, (1978) analyzed ultimate load and stiffened compression plates design, an experimental and theoretical studies into the collapse behavior of box girder compression flanges and to their design are discussed. Special features of flange with stiffeners are identified. Also, several recently proposed design methods are examined for their treatment of these features [3]. In 2014 Tran et al. analyzed the buckling of stiffened curved plates subjected to uniform axial compression stresses. Plates with stiffeners have been used for the bridges construction, for example in box-girder compression flanges. They found that the use of the stiffener is important to resist buckling in the compression plates, also they found that the curvature effect increases the critical buckling resistance [4]

This paper aims to investigation design of bottom flanges in the zones of negative moment at interior piers of continuous spans. To stiffen these areas, longitudinal stiffeners are installed at the bottom flange inside the steel box girder. The longitudinally stiffened plate members in the compression flange provide an economical structure by effectively proportioning the material to resist the produced compressive forces, they generally yield a lightweight structure.

In the AASHTO LRFD [5], the minimum value of longitudinal stiffeners (I_o) is given by:

$$I_o = \alpha w t_f^3 \quad (1)$$

Where:

$\alpha = 0.125 k^3$ for n value equal to 1 and $\alpha = 0.07 k^3 n^4$ for $n > 1$, and k is buckling coefficient, which less than 4, n is the number of longitudinal stiffeners, t_f is the bottom flange thickness and w is the distance between the stiffeners or the spacing between the web and the nearest longitudinal stiffener, when ($n \leq 2$) then the equation will give reasonable required value for I_o . When the longitudinal stiffeners number (n) becomes large, then the values of I_o will be unreasonably large [5].

In 2001 from Yoo research [6] nonlinear regression analysis found that the minimum moment of inertia (I_o) that is needed to ensure an anti-symmetric ($I \geq I_o$) mode shape with the correlation coefficient ($R > 0.95$):

$$I_o = 0.3 \alpha^2 \sqrt{n} w t_f^3 \quad (2)$$

Where, aspect ratio $\alpha = a/w$, and a is panel length.

ANSYS program has been used to examine the compression flange through the elastic buckling analysis.

2. The moment of inertia (I_o) required for anti-symmetric mode shape

The distances between the longitudinal stiffeners at the bottom flange should be equal and the minimum yield strength of the stiffeners must be larger than the minimum yield strength of the bottom plate to which they are connected [5].

Figure 1 shows box girder with longitudinal stiffeners at their bottom flange.

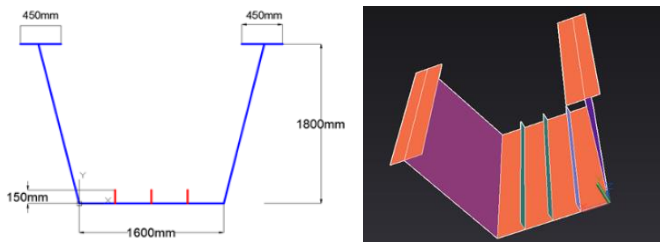


Fig. 1 Girder cross section with flat-bar longitudinal stiffeners.

The stiffened area between two transverse stiffeners is assumed to be applied to a uniformly compressive stress. For a rectangular plate subjected to a uniformly distributed compressive stress, the critical elastic buckling stress is defined by [6].

$$F_{cr} = \frac{K \pi^2 E}{12(1 - \nu^2)} \left(\frac{t_f}{w} \right)^2 \quad (3)$$

Where, K is coefficient of elastic buckling of plate depending on conditions of the edges support and the length to width ratio of the plate. E is the steel modulus of elasticity, ν is Poisson's ratio.

For a square compression plate simply supported along the two edges parallel to the pressure's line, the minimum value of K is equal to 4.00 and 6.97 for a square compression plate clamped along the two edges parallel to the pressure's line [6]. Generally, the connection between the webs and bottom flange is closer to be simply supported, so that K value will equal to 4 [7].

Two types of mode shapes buckled system have been formed due to symmetry of the stiffened plate. For the first type is symmetric form with deflected stiffener which occurs when the stiffeners moment of inertia (I) is below than required moment of inertia (I_o) and the second type is an anti-symmetric form where the stiffeners moment of inertia (I) is larger than (I_o) and the stiffener axis remains straight. These two types of buckling mode shapes are shown in Figs. 2(b) and 2(c).

In case of anti-symmetric buckling, the critical stress (F_{cr}) is not affected by the (I) value, the buckling strength of the stiffened plate does not increase even when the (I) value increased higher than (I_o) value [1].

Figure 3 shows the buckling modes of three stiffeners that divide the plate into four parts with equal spacing. By increasing the value of (I), the number of half-waves will change from 1 (symmetric configuration) to 4 (anti-symmetric configuration), which equals the stiffeners number plus 1 [1].

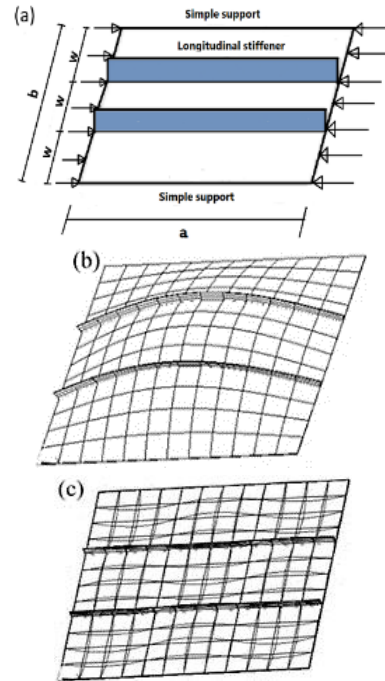


Fig. 2 Buckling mode shapes for $a = 3, n = 2$, (a) boundary condition and loading, (b) Symmetric configuration, (c) Anti-symmetric configuration.

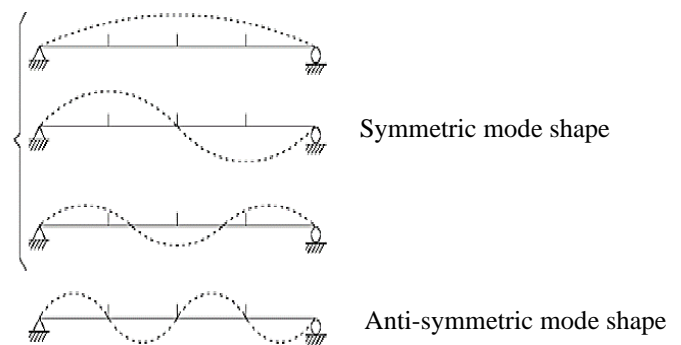


Fig. 3 Different buckling mode shape.

3. Exploratory modeling

The bridge studied is located at Al Terbia intersection in Basra city. The bridge consists of two trapezoidal box girders for continuous bridge with two span and with a radius of curvature of 150 m at the centerline of the cross-section. The total length of bridge equal to 80 m. The concrete deck has 9.5 m width and thickness of 0.25 m, as shown in Fig. 4. The internal bracing is spaced every 4 m. L100 × 100 × 10 angle section members were used for top lateral bracing and internal K-frame. L125 × 125 × 12.5 angle sections were used for the strut beams. In order to study longitudinal stiffeners, single box girder under the fresh concrete weight that can be found by multiplying the concrete density by the deck cross section area (15.5 kN/m on each flange) is used.

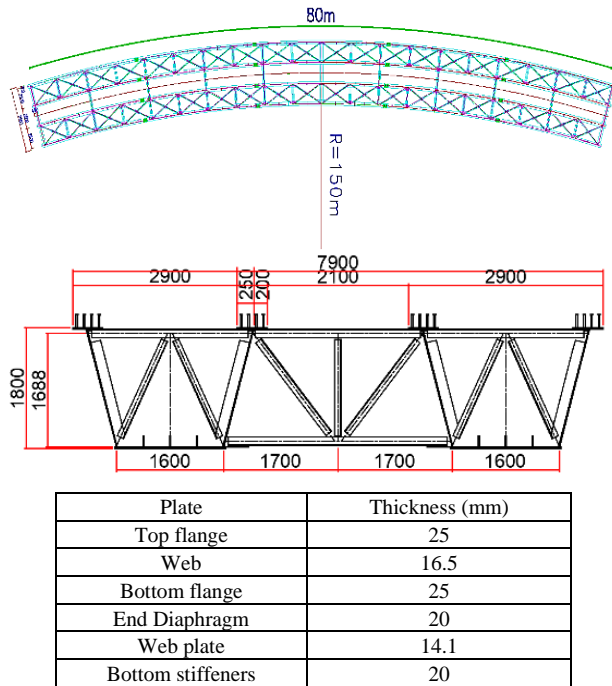


Fig. 4 Details of the box girder bridge.

The purpose of implementing finite element analysis to numerically extract data in this study was to have an uncomplicated method compared with the analytical solutions; therefore, the finite element numerical analysis is used to analyze many different parameters that affect the moment of inertia and critical stress. The stiffened compression plate with simply supported along the two edges parallel to the applied pressure was modeled and analyzed by using finite element modelling (ANSYS19.2) program. SHELL181 elements are used for the simulation both compression flange and stiffeners. Linear elastic buckling is used. The models used realistic dimensions as the compression plate thickness (t_f) is varied from 16 to 30 mm, the longitudinal stiffeners number (n) are varied from 1 to 3, and the length of compression plate is varied from 1 m to 6 m so the aspect ratio varied between 2.5 to 15. Different stiffeners cross section. The buckling analysis of the stiffened plate was conducted using Eigenvalue buckling analysis. Eigenvalue buckling analysis reveals the buckling load factor that has to be multiplied by the applied loads to reach buckling point. ANSYS provides a ready-to-use tool for eigenvalue buckling analysis and calculates buckling load factor along with buckling mode shapes. Note that displacements shown by ANSYS in these analyses have no physical meaning by themselves, and they are only useful in showing the buckling mode shape.

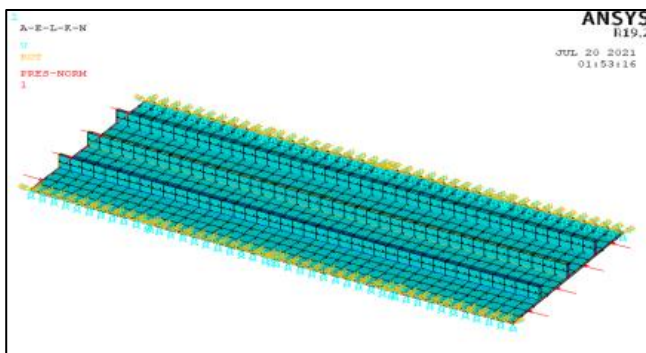


Fig. 5 Loading and boundary condition of stiffened compression plate.

3.1. The aspect ratio of subpanels (a/w)

Table 1 explains the effect of the aspect ratio (α) on the minimum moment of inertia (I_o) that is given by Equation (2) and on the critical stress in the plate. These data represent models for a bottom flange of Basrah bridge girder with different (α) values. Through changing the panel length, it is found that the aspect ratio has a large effect on the minimum required moment of inertia and on the critical stresses.

Table 1. Compression bottom flange of box girder with different (α), ($n = 3$, $t_f = 25$ mm)

α ($\frac{a}{w}$)	A (m)	W (mm)	A_{st} (mm ²)	I_s (mm ⁴) ($\times 10^7$)	I_o (mm ⁴) ($\times 10^7$)	F_{cr} (Eq.) (MPa)	F_{cr} = P_{cr}/A_t (FEA) (MPa)	Dif. (%)	first buckling load (N/mm) (first mode)	Von- Mises Stresses (MPa)
2.5	1	400	150×25	3×2.8	2.03	2828.64	2811.3	-0.61	3741.51	238.89
5	2	400	150×25	3×2.8	8.19	2828.64	2778.6	-1.77	6717.2	243.07
7.5	3	400	150×25	3×2.8	18.268	2828.64	2739.6	-3.15	10074	174.53
10	4	400	150×25	3×2.8	32.476	2828.64	2631.5	-6.97	11989	150.05
12.5	5	400	150×25	3×2.8	50.744	2828.64	2662.5	-5.87	12166	131.79
15	6	400	150×25	3×2.8	73.071	2828.64	2655.0	-6.14	12192	138.54

As can be seen in Fig. 6, the effect of the aspect ratio on the first buckling load is that by increasing panel length (a) the first buckling load will increase. Note that the critical buckling load does not change much for different values of (I_o) as long as it produces an anti-symmetric mode. Also, from the buckling mode shape shown in Fig. 10(a), it can be observed that by increasing the aspect ratio than (10), the buckling mode shape will be symmetric and this unwanted mode.

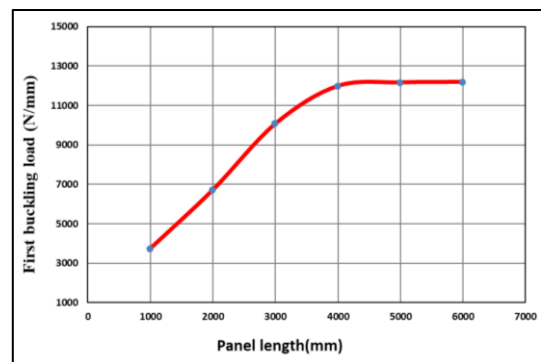


Fig. 6 Effect of panel length on first buckling load.

3.2. Stiffening cross section

A different section of stiffeners has been used as shown in Fig. 7, however the stiffener with flat bar section is the most type utilized in modern designs. Stiffeners can be placed only on one side of the plate (single sided), or on the two sides (double sided).

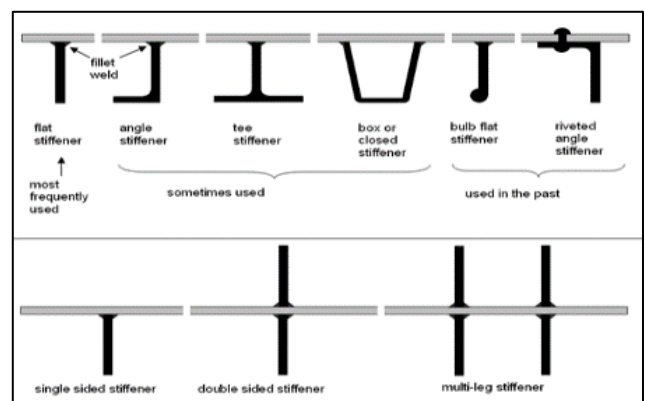


Fig. 7 Stiffener sections.

In general, the moment of inertia of rectangular stiffeners is much less than that of structural (*WT*) of the same total cross-sectional area. Nevertheless, in practice, the use of a flat-bar plate as a longitudinal stiffener is due to its ease in fabrication, maintenance and welding construction. Table 2 shows that critical buckling stress of plate with *WT* and *ST* sections is larger than that for a plate with flat-bar stiffener. However, this increase is not very large as well as the buckling mode shape is similar (ant-symmetric), so it is preferable to use a flat-bar section for ease of dealing with it, see Fig. 10(b).

Table 2. compression bottom flange of box girder with different stiffeners cross sections, $n = 1, w = 800 \text{ mm}, a = 4 \text{ m}$.

α ($\frac{a}{w}$)	A_{stf} (mm^2)	t_f (mm)	I_s (mm^4) ($\times 10^7$)	I_0 (mm^4) ($\times 10^7$)	F_{cr} (eq.) (MPa)	$F_{cr} = P_{cr}/t_f$ (FEA) (MPa)	Dif. (%)	first buckling load (N/mm) (first mode)	Von-Mises Stresses (MPa)
5	150×20	25	2.3	9.375	706.1	742.22	5.1	8662.10	58.68
5	ST9×35	25	3.5	9.375	706.1	759.40	7.5	11603.0	58.05
5	WT13.5×64.5	25	12.617	9.375	706.1	756.96	7.2	11954.9	59.38
5	WT16.5×84.5	25	25.352	9.375	706.1	758.24	7.4	12171.0	57.86

3.3. Flange thickness

Figure 8 illustrates the effect of different bottom flange thicknesses on the critical buckling load. The model (Basrah bridge girder) has a subpanel width (400 mm), three longitudinal stiffeners, an aspect ratio (10), and different flange thicknesses. Table 3 shows that the critical buckling load will increase by increasing the plate thickness which increases the required minimum stiffness for longitudinal stiffeners (I_0). Fig. 10(c) shows that the compression plate with all different thicknesses provides an anti-symmetric buckling mode shape.

Table 3. compression bottom flange of box girder with different thickness, $a = 4 \text{ m}, n = 3, w = 400 \text{ mm}$.

α ($\frac{a}{w}$)	A_{stf} (mm^2)	t_f (mm)	I_s (mm^4) ($\times 3 \times 10^3$)	I_0 (mm^4) ($\times 10^7$)	F_{cr} (eq.) (MPa)	$F_{cr} = P_{cr}/t_f$ (FEA) (MPa)	Dif. (%)	first buckling load (N/mm) (first mode)	Von-Mises Stresses (MPa)
10	(150×25)	16	2.813	8.51	1156.7	1069.3	-7.6	1069.3	125.192
10	(150×25)	18	2.813	12.12	1344.5	1272.2	-5.4	1272.2	161.106
10	(150×25)	20	2.813	16.63	1807.6	1810.5	0.16	1810.5	176.732
10	(150×25)	22	2.813	22.13	2187.2	2183.8	-0.15	2183.8	156.63
10	(150×25)	24	2.813	28.73	2602.9	2478.5	-4.8	2478.5	149.185
10	(150×25)	25	2.813	32.48	2828.6	2631.5	-7.0	2631.5	150.05
10	(150×25)	27	2.813	40.91	3294.4	2959.6	-10.2	2959.6	118.35
10	(150×25)	30	2.813	56.12	3368.0	3344.2	-0.7	3344.2	251.054

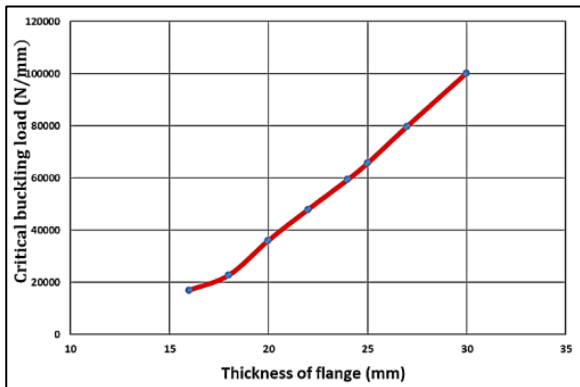


Fig. 8 Critical buckling load versus flange thickness.

3.4. Stiffener numbers (n)

Table 4 demonstrates the effect of the number of stiffeners, (n), on the critical compressive strength and the minimum required moment of inertia for three different cross-sections of stiffeners. Fig. 9 shows that when the number of stiffeners

increases then the critical buckling load also increases. The critical buckling load for a curved compression plate with *ST* and *WT* sections is larger than that for a plate with a flat-bar section. Fig. 10(d) shows the buckling mode for the compression plate with two stiffeners with different cross-sections.

Table 4. compression bottom flange of box girder with different number of stiffeners, $a = 4 \text{ m}, t_f = 25 \text{ mm}$.

α ($\frac{a}{w}$)	N	W (mm)	A_{stf} (mm^2)	I_s (mm^4) ($\times 10^7$)	I_0 (mm^4) ($\times 10^7$)	F_{cr} (eq.) (MPa)	$F_{cr} = P_{cr}/t_f$ (FEA) (MPa)	Dif. (%)	first buckling load (N/mm) (first mode)	Von-Mises Stresses (MPa)
5	1	800	(150×20)	2.3	9.375	706.1	742.2	5.1	8662.1	58.68
6.7	2	600	(150×20)	4.6	17.85	1255.3	1439	14.6	11021	120.5
10	3	400	(150×20)	6.9	32.476	2828.6	2523	-10.8	11346	110.04
5	1	800	ST9×35	3.5	9.375	706.1	759.4	7.5	11603	58.05
6.7	2	600	ST9×35	7	17.85	1255.3	1650	31.4	13826	107.2
10	3	400	ST9×35	10.5	32.476	2828.6	2978	5.3	14230	232.7
5	1	800	WT12×27.5	4.75	9.375	706.1	753.83	6.7	11644	58.32
6.7	2	600	WT12×27.5	9.5	17.85	1255.3	1588	26.5	13518	39.9
10	3	400	WT12×27.5	14.25	32.476	2828.6	2800	-1.0	12135	142.79

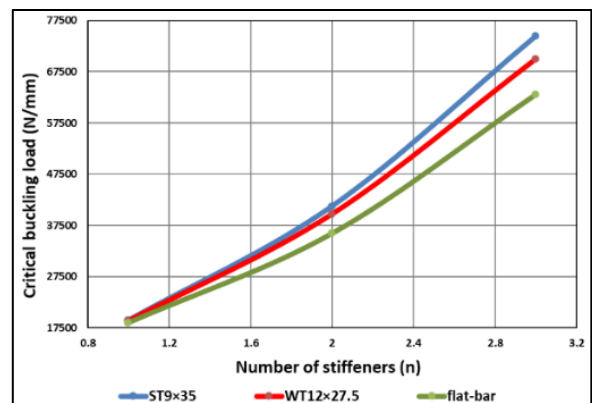
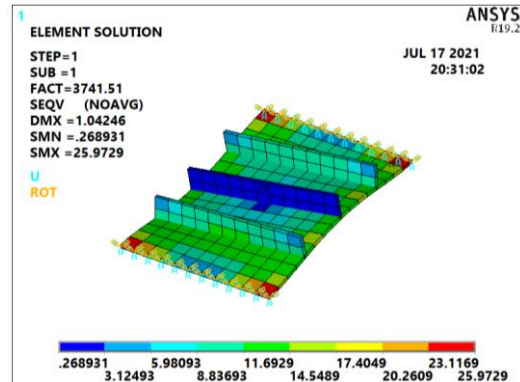
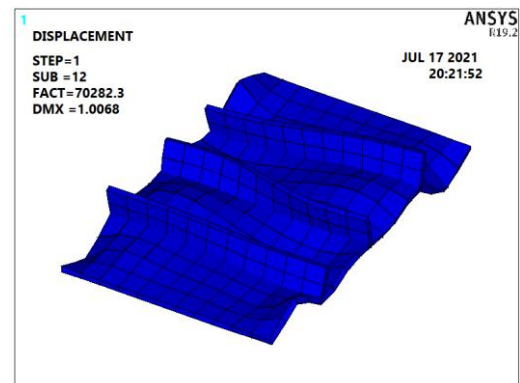


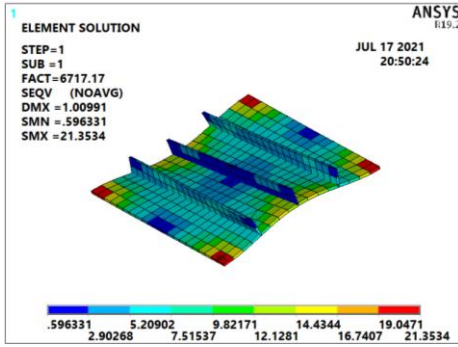
Fig. 9 Critical buckling load for plate with different number and cross sections.



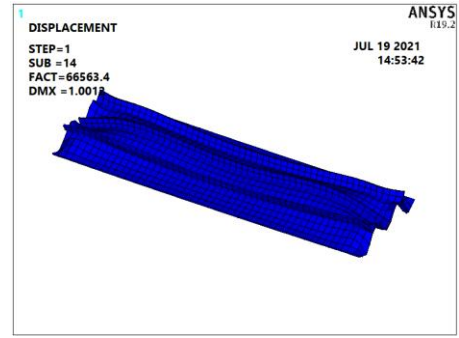
First buckling load for compression plate with $\alpha = 2.5$.



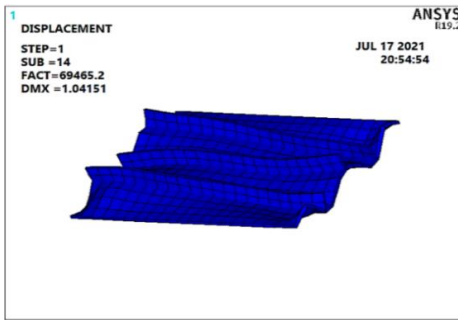
Critical buckling load for compression plate with $\alpha = 2.5$.



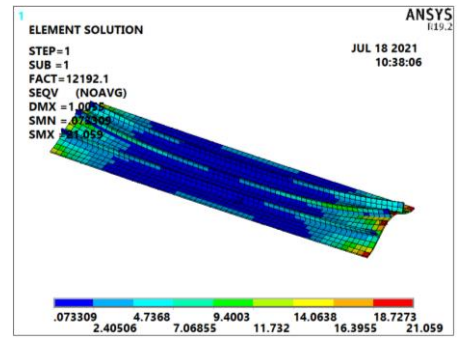
First buckling load for compression plate with $\alpha = 5$.



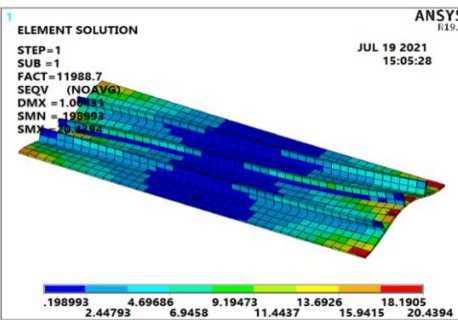
Critical buckling load for compression plate with $\alpha = 12.5$.



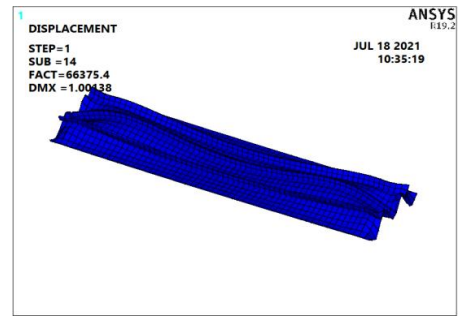
Critical buckling load for compression plate with $\alpha = 5$.



First buckling load for compression plate with $\alpha = 15$.

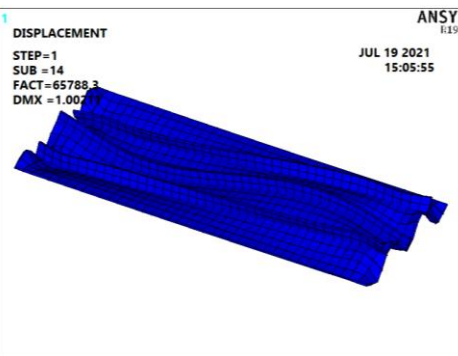


First buckling load for compression plate with $\alpha = 10$.

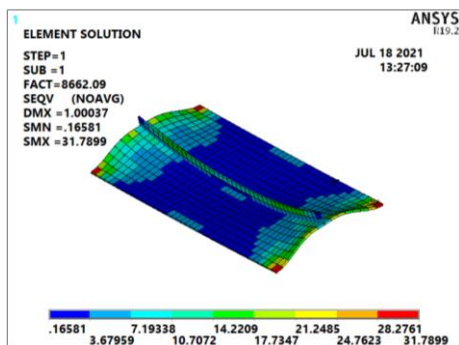


Critical buckling load for compression plate with $\alpha = 15$.

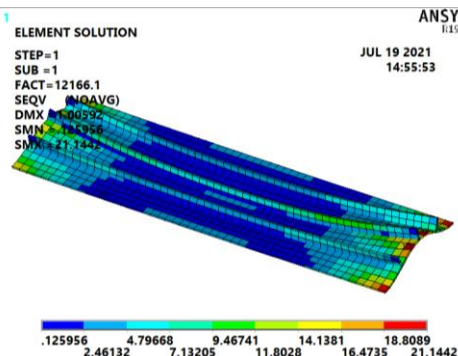
(a) First and critical buckling load of the panel plate with different aspect ratio.



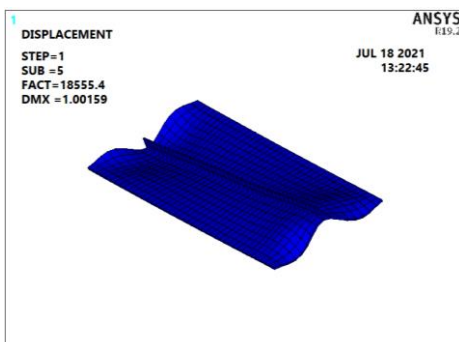
Critical buckling load for compression plate with $\alpha = 10$.



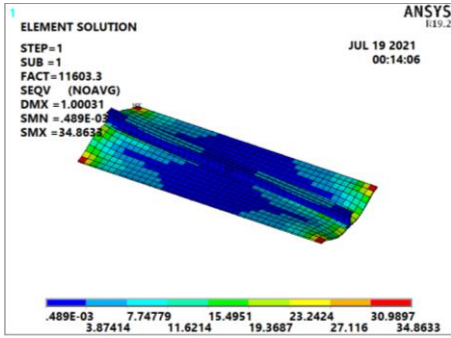
First buckling load for compression plate with flat-bar section.



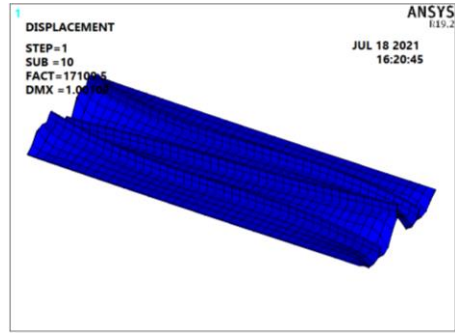
First buckling load for compression plate with $\alpha = 12.5$.



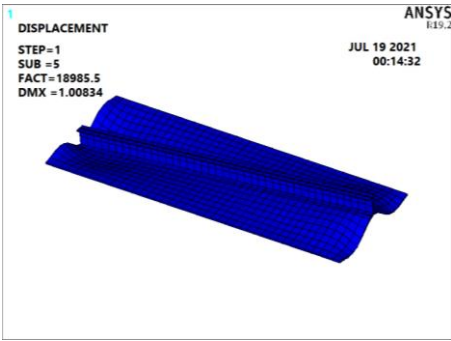
Critical buckling load for compression plate with flat-bar section.



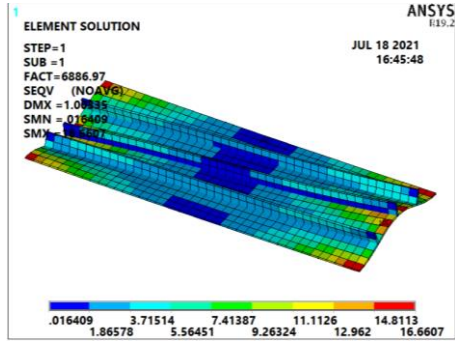
First buckling load for compression plate with ST 9×35 section.



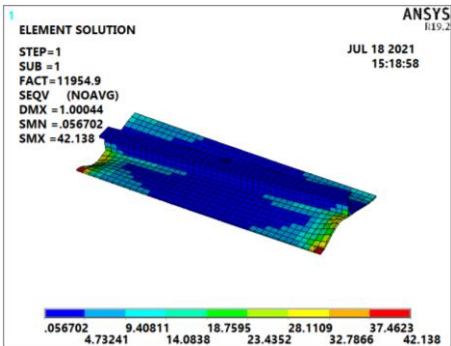
Critical buckling load for compression plate with $t_f = 16$ mm.



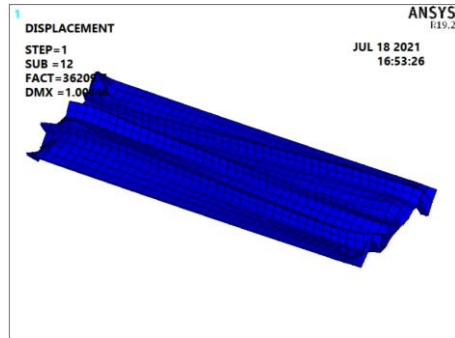
Critical buckling load for compression plate with ST 9×35 section.



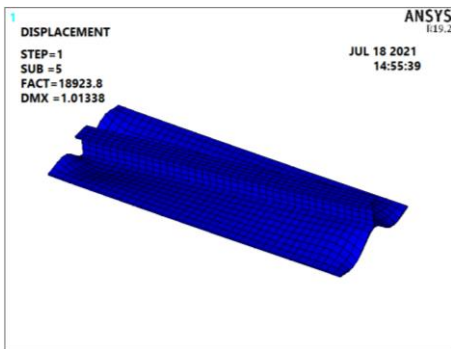
First buckling load for compression plate with $t_f = 20$ mm.



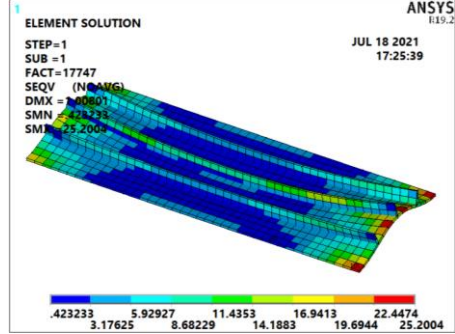
First buckling load for compression plate with WT 13.5×64.5 section.



Critical buckling load for compression plate with $t_f = 20$ mm.

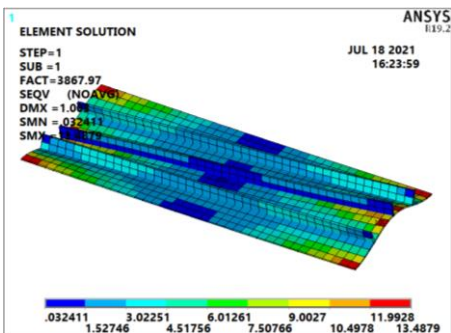


Critical buckling load for compression plate with WT 13.5×64.5 section.

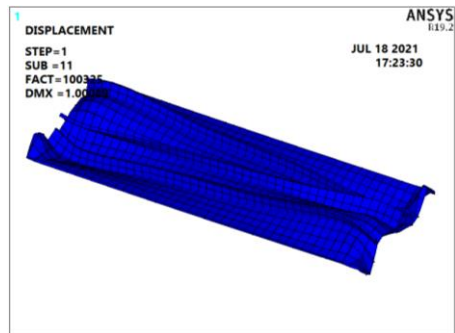


First buckling load for compression plate with $t_f = 30$ mm.

(b) First and buckling load of the panel plate with different section of stiffeners.

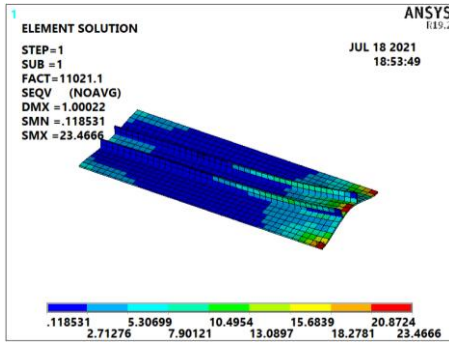


First buckling load for compression plate with $t_f = 16$ mm.

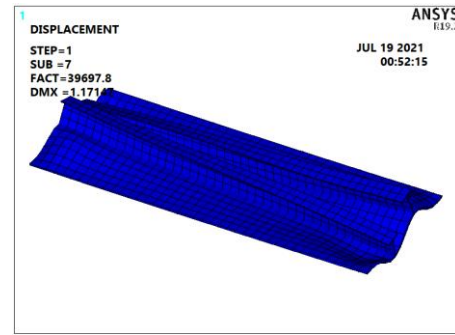


Critical buckling load for compression plate with $t_f = 30$ mm.

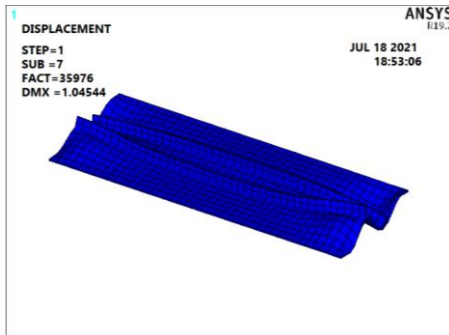
(c) First and critical buckling load of the panel plate with different plate thickness.



First buckling load for compression plate with two stiffeners of flat-bar section.



Critical buckling load for compression plate with two stiffeners of WT 12 × 27.5 section.



(d) First and critical buckling load of the panel plate with different sections of stiffener.

Fig. 10 Buckling mode shapes of the stiffened compression plate by ANSYS.

4. Longitudinal stiffener effect on the box girder

In 1973, Heins et al. [8] predicted that the contribution of longitudinal stiffener to the moment of inertia of steel structure was less than 1 %. However, this stiffeners contribution must be taken into account because of the shifting of the centroidal axis. Lateral bending stresses occurred in the flange of stiffeners due to the bridge curvature, these stiffener flanges share the flanges of girder in carrying a stress f_s and preventing bending moments, as shown in the Fig. 11, the stress is given by:

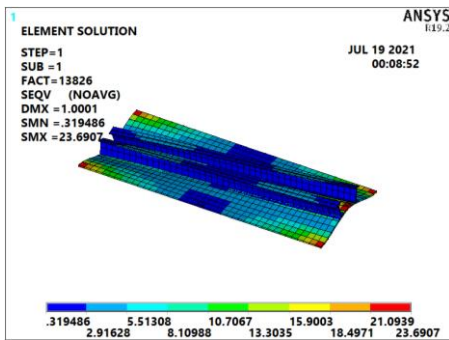
$$f_s = \frac{(y_b - y_s) f_b}{y_b} \tag{4}$$

Where,

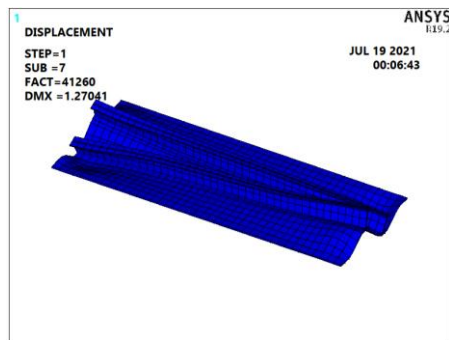
f_b = max bending stress (MPa), in bottom flange of girder.

y_b = distance from neutral axis to bottom flange (mm).

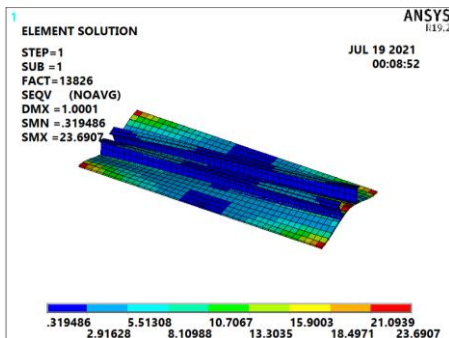
$y_b - y_s$ = distance from neutral axis to top flange of stiffener (mm).



First buckling load for compression plate with two stiffener of ST 9 × 35 section.



Critical buckling load for compression plate with two stiffener of ST 9 × 35 section.



First buckling load for compression plate with two stiffeners of WT 12 × 27.5 section.

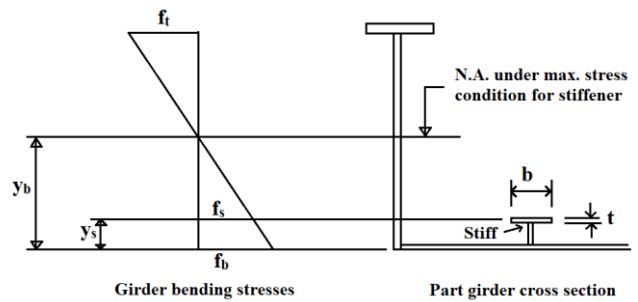


Fig. 11 Bending stress in longitudinal stiffeners.

The stiffeners flange is subjected to a lateral bending moment due to its curvature.

$$M_{LC} = \frac{f_s b t d^2}{10 R} \tag{5}$$

Where,

b = flange width of stiffener (mm),

t = flange thickness of stiffener (mm),

d = unbraced length (mm),

R = radius of curvature (mm).

The corresponding lateral bending stress is [8].

$$f_{wc} = \frac{6 f_s d^2}{10 R b} \tag{6}$$

In the present analysis the finite element program ANSYS 19.2 is used, beam elements are used for modelling the longitudinal stiffeners. In order to study the stiffeners effect on bottom plate stresses, box girder with or without the longitudinal stiffener were used, both models have the same geometric configuration. Tables 5 and 6 show the stresses for the two models in the max + M and max - M area respectively. Fig. 12 shows the node numbering of the bottom flange of box girder.

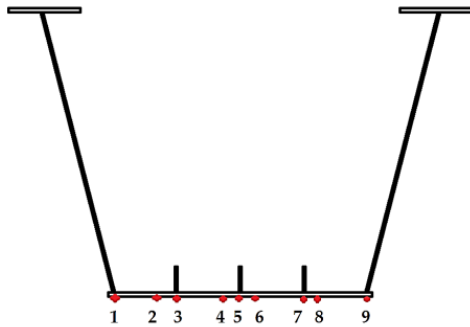
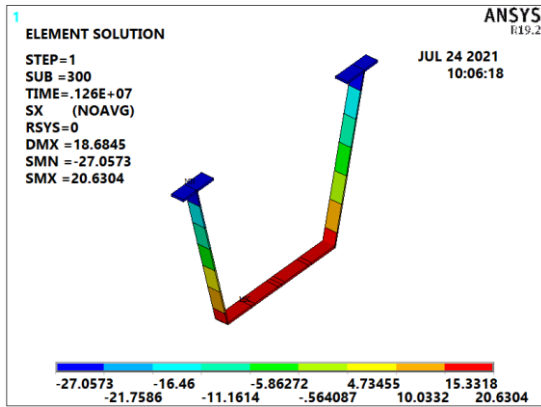
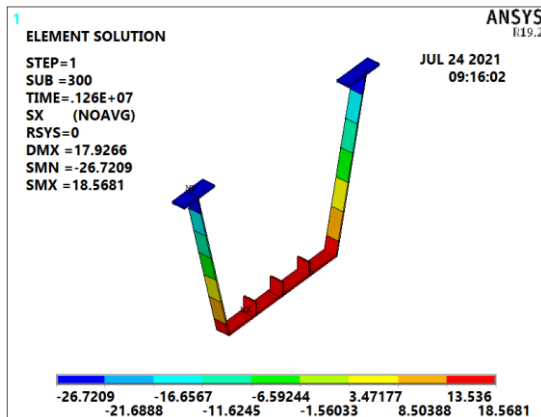


Fig. 12 Node numbering on a positive and negative bending moment region (all dimensions in mm).

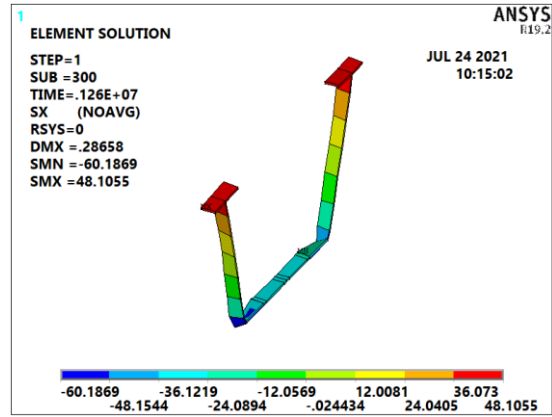


(a) Stresses in positive bending moment region for girder without longitudinal stiffeners (MPa).

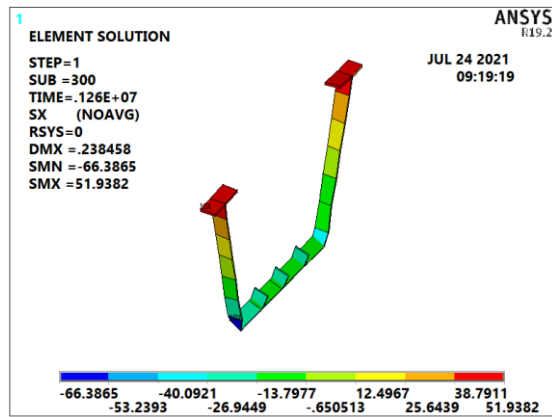


(b) Stresses in positive bending moment region for girder with longitudinal stiffeners (MPa).

Fig. 13 Stresses in positive bending moment region.



(a) Stresses in negative bending moment region for girder without longitudinal stiffeners (MPa).



(b) Stresses in negative bending moment region for girder with longitudinal stiffeners (MPa).

Fig. 14 Stresses in negative bending moment region.

Table 5. Bottom flange stresses in Max + M region.

Location along bottom flange	Stresses in BF with longitudinal stiffener(MPa)	Stresses in BF without long stiffener (MPa)	Difference between stresses With and without long. Stiff (%)
1	16.961	18.821	0.109
2	17.902	19.866	0.110
3	17.191	19.97	0.162
4	18.069	20.111	0.113
5	17.392	20.206	0.162
6	18.223	20.304	0.114
7	17.646	20.457	0.159
8	18.512	20.575	0.111
9	17.709	19.63	0.108

Table 6. Bottom flange stresses in Max - M region.

Location along bottom flange	Stresses in BF with longitudinal Stiffener (MPa)	Stresses in BF without long stiffener (MPa)	Difference between stresses with and without long. Stiff (%)
1	-52.211	-60.215	0.153
2	-45.083	-54.58	0.211
3	-27.482	-45.152	0.643
4	-37.484	-51.126	0.364
5	-30.535	-53.593	0.755
6	-37.611	-51.593	0.372
7	-28.196	-45.929	0.629
8	-47.069	-55.469	0.178
9	-49.309	-57.228	0.161

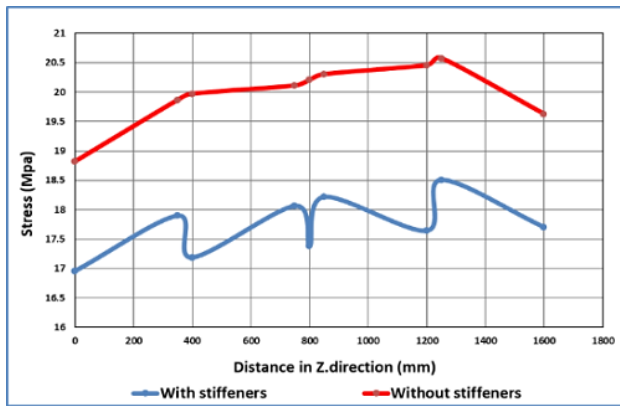


Fig. 15 Comparison curves of bottom flange stresses in Max + M region.

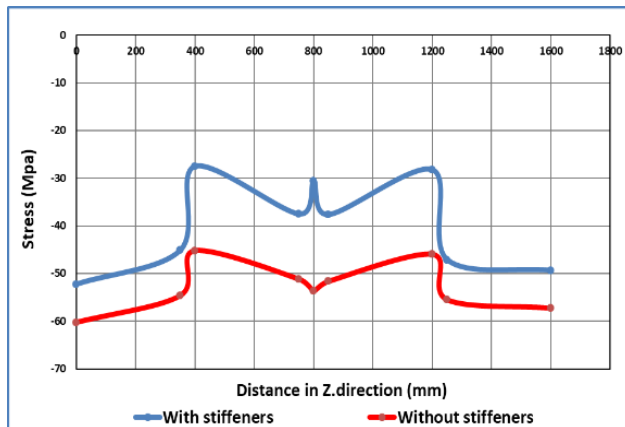


Fig. 16 Comparison curves of bottom flange stresses in Max - M region.

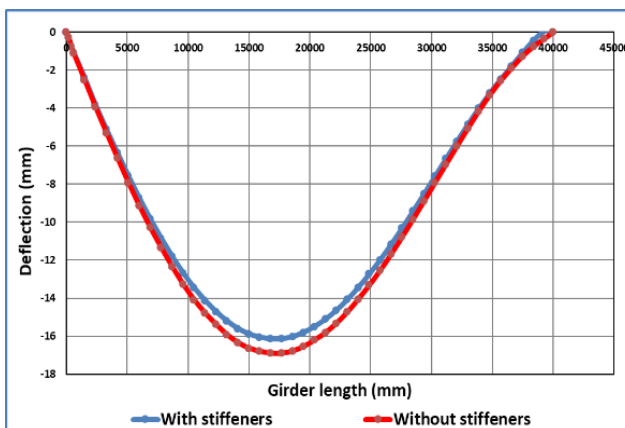


Fig. 17 Deflection along the center line of nodes in the bottom flange of box girder.

From the last results it can be noted that the stiffeners have larger effect on the stresses at the negative moment region than on the stresses of positive and negative moment regions as can be seen in ANSYS models in Figs. 13 and 14, this two Figures show that the bottom flange stresses of the steel box girder with longitudinal stiffeners is less than that for box girder without these stiffeners, the larger difference was at the location of longitudinal stiffeners at (400 mm, 800 mm and 1200 mm). Figure 17 shows that the deflection of box girder without stiffeners is larger than deflection of girder with stiffeners.

5. Conclusions

The compression plate panel with simply supported along the two edges parallel to the applied pressure was modeled and analyzed by ANSYS 19.2 software. In the current study, the minimum required stiffness of longitudinal stiffeners for the bottom flange of box girder has been investigated.

Depended on the obtained results of the investigated models, the following conclusions are made:

1. Aspect ratio (α) has a significant effect on the first buckling load, by increasing the length of the plate (a), the first buckling load will increase. However, by increasing the aspect ratio than 10 the buckling mode shape will be symmetric.
2. There is no large difference in critical buckling stress between the plate with flat bar section or with other sections, therefore, it's preferable to use a flat bar plate due to their many advantages.
3. The results showed that the increase of panel plate thickness leads to increase the minimum moment of inertia and the critical buckling load.
4. The critical buckling load increased by increasing the number of stiffeners.
5. Placing longitudinal stiffeners has a large effect on the stresses of compression plate of negative moment region than the stresses of compression plate of positive moment region.

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